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A viscoelastic, viscoplastic model of cortical bone valid at low and high strain rates

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ABSTRACT

The stress–strain behavior of cortical bone is well known to be strain-rate dependent, exhibiting both viscoelastic and viscoplastic behavior. Viscoelasticity has been demonstrated in literature data with initial modulus increasing by more than a factor of 2 as applied strain rate is increased from 0.001 to 1500 s^{-1}. A strong dependence of yield on strain rate has also been reported in the literature, with the yield stress at 250 s^{-1} having been observed to be more than twice that at 0.001 s^{-1}, demonstrating the material viscoplasticity. Constitutive models which capture this rate-dependent behavior from very low to very high strain rates are required in order to model and simulate the full range of loading conditions which may be experienced in vivo; particularly those involving impact, ballistic and blast events. This paper proposes a new viscoelastic, viscoplastic constitutive model which has been developed to meet these requirements. The model is fitted to three sets of stress–strain measurements from the literature and shown to be valid at strain rates ranging over seven orders of magnitude.

1. Introduction

Cortical bone, a major structural component of the human body, functions in both a load-bearing and a protective capacity. As a material, bone has been the subject of mechanical investigation and characterization for over 150 years [1], and the rate-dependent nature of its behavior has been documented in numerous studies [2–11]. Most of this work has focused on testing and modeling cortical bone at strain rates physiologically relevant to the activities of walking and running, in the range of 0.001–0.1 s^{-1} [12]. This paper, however, focuses on developing a constitutive model capable of capturing the mechanical behavior of cortical bone over the full range of strain rates relevant to impact loading conditions (0.001–1000 s^{-1}). We begin with a review of previously published data relevant to the development of such a model.

One of the earliest and most widely cited high-strain-rate studies was undertaken by McElhaney and employed a Tinius Olsen electromatic testing machine, in combination with a novel air-gun machine, to characterize bovine and embalmed human cortical bone samples from the femur in uniaxial compression at rates ranging from 0.001 to 1500 s^{-1} [2]. The resulting stress–strain curves for human cortical bone (Fig. 1) have served as a benchmark for well over 40 years and have subsequently found widespread use in the validation and calibration of potential cortical bone constitutive models [13–15].

A subsequent study by Wood employed Instron TT-C and Plastechon hydraulic testing machines to test human cranial bone in tension at strain rates of 0.005–150 s^{-1} [4]. The resulting initial elastic moduli ranged as a function of strain rate from 10.3 to 22.1 GPa. A regression line of this data fitted by the author gave the following result:

$$E = [16.0 + 1.93\log(\dot{\varepsilon})]\ \text{GPa}. \tag{1}$$

This equation, while providing a good fit for the rates within the study, incorrectly predicts a negative elastic modulus at extremely low strain rates.

The split-Hopkinson pressure bar (SHPB) technique was used by Tennyson et al. to study the effect of post-mortem age on bovine femoral cortical bone tested in compression at rates of 10–450 s^{-1} [5]. The initial elastic moduli as a function of strain rate for samples with a post-mortem age of 11 days was then fit to a Voigt model with the following form:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}, \tag{2}$$

where the long-term elastic constant, E, was found to be 18.1 GPa and the viscosity, η, 2.1×10^4 Pa s.

Another study, conducted by Crowninshield and Pope, employed both an Instron TT-CM1 and a drop hammer device to test bovine femoral cortical bone in tension at strain rates of 0.001–250 s^{-1} [6]. They measured an apparent elastic modulus that increased monotonically with strain rate from 10 to 12.5 GPa and provided a set of characteristic stress–strain curves corresponding to the various strain rates attained.

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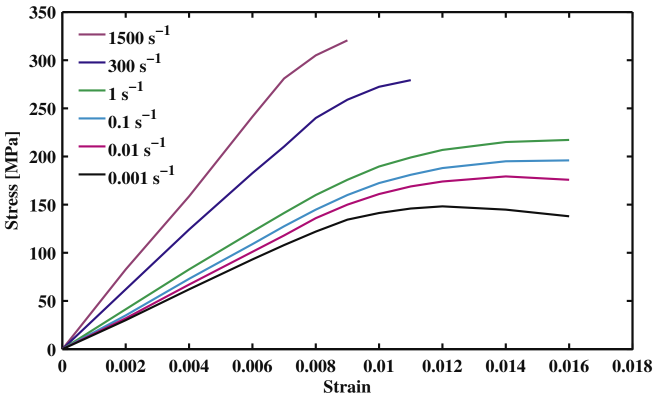

Fig. 1. McElhaney's characteristic compressive stress–strain curves for human femoral cortical bone as a function of strain rate [2].

Lewis and Goldsmith subsequently utilized the SHPB procedure to test bovine femoral cortical bone in compression, tension, torsion, and combined torsion and compression [7]. From the compression data, a linear viscoelastic relaxation function was fitted to the initial stress–strain behavior, giving a time constant of 13 μs and a long-term elastic modulus of 19.3 GPa. To allow for comparison with the work of Tennyson and others, this function was also evaluated as an equivalent Voigt model, which yielded a viscosity of 3.23×10^5 Pa s.

In a study by Melnis and Knets, human femoral cortical bone was tested in tension at various hydration states at strain rates ranging from 10^{-5} to 1 s^{-1} [9]. It was found that the material behaved in a non-linear viscoelastic manner, requiring two time constants to describe the material's initial viscoelastic behavior. The amount of viscoelasticity was observed to decrease significantly with reduction in water content. In addition to fitting their data with a viscoelastic function with two time constants, Melnis and Knets also attempted to capture the plastic behavior of cortical bone by employing a piecewise function with a constant hardening slope above a critical value of strain.

Katsamanis and Raftopoulos employed a universal testing machine and SHPB theory to test, in both tension and compression, human femoral cortical bone that had been allowed to dry for several days [10]. Their results showed an average static (strain rate of $2 \times 10^{-5}\ s^{-1}$) Young's modulus of 16.2 GPa and an average dynamic (strain rate of 100 s^{-1}) Young's modulus of 19.9 GPa. A Voigt viscoelastic model fitted to the data of this study gave a viscosity of 3.7×10^4 Pa s.

Finally, in one of the most recent studies, uniaxial compression tests were conducted by Adharapurapu et al. at a variety of strain rates (ranging from 10^{-3} to $10^3\ s^{-1}$) on bovine femoral cortical bone, in both a dry and a rehydrated state [11]. The resulting initial elastic moduli for the dry specimens ranged from 5.3 to 20 GPa as a function of strain rate, while those of the rehydrated specimens ranged from 5 to 30 GPa, providing an interesting insight into the role of hydration in the high-strain-rate behavior of cortical bone.

A summary of the various apparent Young's moduli found in these studies is provided as a function of strain rate in Fig. 3. Within the literature a Voigt model is often used to characterize the viscoelastic behavior of cortical bone. The resulting viscosity values range widely from 2.1×10^4 to 2.3×10^9 Pa s with the corresponding long-term elastic constants ranging from 11 to 19.3 GPa [4–10,16,17]. It has been shown, however, that a linear viscoelastic theory with only one relaxation mechanism does not apply to bone over the wide range of strain rates of interest in this study [9,13]. This modeling approach, while useful in understanding the most basic viscoelastic properties of bone, and allowing for comparison amongst experimental results, does not adequately capture the strain rate dependence over the wide range of rates need for robust loading simulations (0.001–1000 s^{-1}) and moreover neglects the yield and post-yield behavior of the material entirely.

In an attempt to capture both the viscoelastic and viscoplastic characteristics of bone, Hight and Brandeau proposed employing a modified version of the Ramberg–Osgood equation [14]:

$$\varepsilon = \frac{\sigma}{c\dot{\varepsilon}^d} + a\sigma^N\dot{\varepsilon}^b. \tag{3}$$

Here the first term captures the initial viscoelastic behavior and the second term accounts for the viscoplastic rollover in stress. The model parameters (a, b, c, d, N) were fitted to three sets of stress–strain curve data found in the literature (McElhaney [2], Wood [4], and Crowninshield and Pope [6]). The Hight and Brandeau model provides a reasonable curve fit for the Wood and Crowninshield and Pope data sets, but the curve fits for the McElhaney data demonstrate a significant level of error, especially at strain rates of 300 and 1500 s^{-1}. Furthermore, the parameters in the curve-fitting process varied greatly amongst data sets (by nearly 90 orders of magnitude in the case of parameter a) and are therefore unlikely to be capturing the underlying mechanisms of deformation.

The previous attempts at capturing the viscoelastic and/or viscoplastic behavior of cortical bone have achieved some limited success, each within some confined regime of loading. The nature of impact, ballistic and blast loading scenarios, however, requires a model that spans a wide range of strain rates. Here, we develop a constitutive model which captures the mechanical behavior from very low to very high strain rates and which is also extended to three dimensions to enable incorporation into finite-element simulation of inhomogeneous boundary value problems typical of impact-type loading events.

2. Model development

The proposed model framework is shown in the rheological schematic of Fig. 2, which separates the viscoelastic and viscoplastic contributions of the material response. The development and behavior of each modeling component is provided in the following sections, followed by a summary of the model behavior when the two components are combined.

2.1. Viscoelastic component

To gain further insight into the nature of cortical bone viscoelasticity, we first examine the initial elastic moduli of the previously discussed data sets against strain rate (Fig. 3). Despite the variety of testing conditions (both compression [C] and tension [T] on hydrated [W], embalmed [E], and dehydrated [D] specimens) and the variety of species tested (both bovine [B] and human [H]), the overall trend between the initial apparent Young's modulus and the log

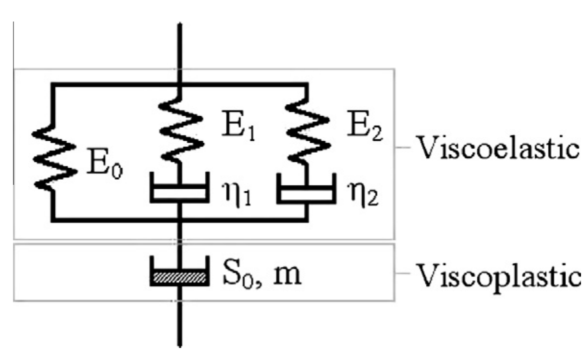

Fig. 2. Schematic representation for the mechanical analog of the viscoelastic, viscoplastic constitutive model proposed for cortical bone.

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