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The use of flat punch indentation to determine the viscoelastic properties in the time and frequency domains of a soft layer bonded to a rigid substrate

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Abstract

This paper reports a computational study of the indentation of a flat punch into a compressible elastic layer (with Poisson's ratio varying from 0 to 0.49) bonded to a rigid substrate. Based on the computational results and using Sneddon's solution [Sneddon IN. The relation between load and penetration in the axisymmetric Boussinesq problem for a punch of arbitrary profile. Int J Eng Sci 1965;3:47] and the asymptotic solution [Jaffar MJ. A general solution to the axisymmetric frictional contact problem of a thin bonded elastic layer. Proc Inst Mech Eng C 1997;211:549; Yang FQ. Asymptotic solution to axisymmetric indentation of a compressible elastic thin film. Thin Solid Films 2006;515:2274] as the two limits, a simple expression of the load–depth curve valid for an arbitrary ratio of the indenter radius to the thickness of the layer is obtained. Further, a correlation between indentation load and depth for a rigid flat punch indenting into linearly viscoelastic layers bonded to a rigid substrate is proposed by using the correspondence principle. Several procedures are suggested based on the results reported in this study to determine the viscoelastic properties of the layer in the time or frequency domains. The findings are verified by numerical examples. The results may facilitate the use of depth-sensing indentation tests to characterize the mechanical properties of polymeric films or functional coatings on hard substrates, and some biological materials, e.g. articular cartilage.

Keywords: Indentation; Biological materials; Polymeric films and coatings; Relaxation function; Viscoelastic properties

1. Introduction

Depth-sensing instrumented indentation tests are widely used to determine the local mechanical properties of materials at different scales, e.g. Young's modulus and the hardness [1,2], yield strength and/or strain-hardening exponent of metals (e.g. [3–7]). Recently, there has been growing interest in the application of indentation tests to determine mechanical properties of materials with time-dependent deformation behaviour including polymers [8–15], foods [16] and biological tissues [17–23]. The main concern of the present work is the axisymmetric contact problem of a circular flat punch indenting into an elastic or viscoelastic layer bonded to a rigid substrate, which is schematically shown in Fig. 1. The solution to this problem may yield applications in many circumstances, such as using indentation tests to determine elastic or viscoelastic properties of articular cartilage and polymeric films or functional coatings on hard substrates. During the past decades, this problem has been addressed in numerous publications (e.g. [24–30]). When the radius of the indenter a is much smaller than the layer thickness t_0, the problem degenerates to that of a flat punch indenting into a flat half-space. The corresponding load–depth relation may be obtained from Sneddon's solution [31]. In the case that a is much larger than t_0, Jaffar [28] and Yang [30] have reported the asymptotic solution to the contact problem from which the load–depth relation can be found. However, a big gap still exists between Sneddon's solution [31] and the asymptotic solu-

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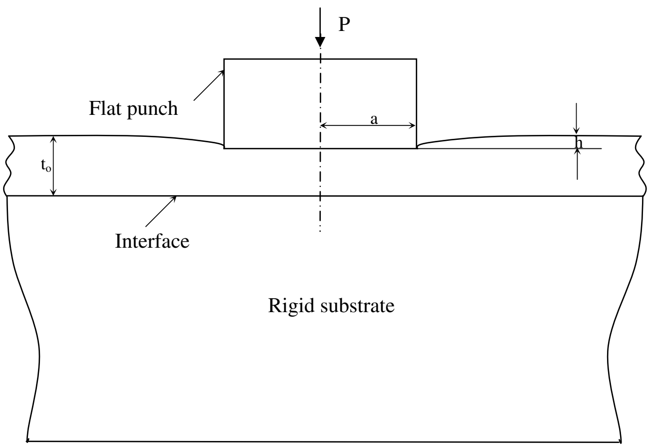

Fig. 1. A schematic diagram of a flat punch indenter indenting into a soft layer bonded to a rigid substrate.

tion [28,30], for which the development of analytical solutions appears to be impossible, suggesting that the problem has to be solved numerically. Based on the technique proposed by Lebedev and Ufliand [32], Hayes et al. [24] reported explicit expressions of the load–depth curve for Poisson's ratios of 0.3–0.5 and a/t_0 ratios in the range 0.2–8. Jaffar proposed a simple numerical technique [27] which can be used to solve the problem when a/t_0 ratio is in the range of 0–20.

The present study starts from the development of a simple expression for the correlation between the indentation load and the indentation depth in terms of the material properties and the geometry of both the indenter and the layer. This relationship is the key for the determination of the elastic or linearly viscoelastic properties of the layers from depth-sensing instrumented indentation tests using a circular flat punch. As mentioned above, for some specific given ratios of the indenter radius a to the layer thickness t_0, the load–depth relationship is available in the literature [24]. However, to the best of the authors' knowledge the correlation valid for an arbitrary a/t_0 ratio has not been reported. Furthermore, based on the results presented in the present work and recent advances in the understanding of the indentation of time-dependent materials [8,10,33,34], several procedures are suggested to determine the relaxation function of a soft layer bonded to a rigid substrate in the time or frequency domains. Numerical simulations are performed to verify the effectiveness of the methods.

2. A cylindrical flat punch indenting into an elastic or linearly viscoelastic layer bonded to a rigid substrate

In the present work, finite-element analysis (FEA) is performed to construct the explicit expression of the load–depth relation valid for an arbitrary a/t_0 ratio, instead of using the numerical techniques in the literature [32,27]. The advantage of using FEA is that the effects of friction can be taken into consideration.

To provide a guideline for constructing the load–depth relation, the theoretical analysis performed by Hayes et al. [24] is invoked, from which the following relation can be obtained:

$$P = \frac{2E}{1-v^2} ah\Pi\left(\frac{a}{t_0}, v\right), \tag{1}$$

where P, E and h are the indentation load, Young's modulus and indentation depth, respectively. v is the Poisson's ratio. Π is a dimensionless function, which is determined via systematic FEA. The commercial finite-element software ABAQUS [35] is used in the present work. The indenter is assumed to be rigid and an axisymmetric four-node element is used to model the indented layer. To guarantee the convergence of the simulation, more than 40 elements are used in the contact region. Taking into consideration that the elastic solution obtained in this section forms the basis of deriving the linearly viscoelastic solution using the correspondence principle [36], the indentation depth is taken to be much smaller than the thickness of the layer and the indenter radius, i.e. $h = 0.03\min(a,t_0)$. The ratio of a to t_0 varies between two limits, i.e. from 0.02 to 150. The lower limit corresponds to the indentation of a half-space, and at the upper limit the load–depth curve is that of the asymptotic solution [28,30]. The Poisson's ratio for the compressible layer changes from 0 to 0.49. The friction coefficient between the indenter and the layer varies from 0.05 to 0.5.

The computational results show that the dimensionless function in Eq. (1), corresponding to different a/t_0 ratios, varies over a wide range, i.e. from around 1 to a value of some thousands. For better representation, the dimensionless function Π is normalized first by using the following equation:

$$f = 1.03\,e^{\left(-\frac{a}{5t_0}\right)} + \frac{\pi}{2}\frac{(1-v)^2}{1-2v}\frac{a}{t_0}\left[1 - e^{\left(-\frac{a}{t_0}\right)}\right], \tag{2}$$

where e is the exponential function. When a/t_0 tends to zero, the function f approaches 1 and Eq. (1) degenerates to Sneddon's solution. When a/t_0 goes to infinity, the load–depth curve give by Eq. (1) converges to the asymptotic solution of Jaffar [28] or Yang [30], and f is given by:

$$f = \frac{\pi(1-v)^2 a}{2(1-2v)t_0}. \tag{3}$$

The normalized dimensionless functions Πs corresponding to two different friction coefficients are given in Figs. 2 and 3. With known Poisson's ratio v and a/t_0 ratio, the dimensionless function Π can be determined from Eq. (2) and Fig. 2 or 3. When v and a/t_0 are not on the curves in Fig. 2 or 3, we suggest using linear interpolation to obtain a solution for the Π/f. The accuracy of the linear interpolation has been carefully examined. For several sets of v and a/t_0, the difference between the values of the Π/f. determined using linear interpolation (open squares in Fig. 2) and those obtained from FEA (open circles in Fig. 2) is less than 4%. A previous study [37] has shown that the effect of friction on the load–depth curve has a close relationship